## KEKULE STRUCTURES AND RESONANCE ENERGIES OF BENZENOID HYDROCARBONS Robert Swinborne-Sheldrake and William C. Hemdon\*

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Institute Rugjer Boskovi6, P.O. Box 1016, Zagreb, Croatia, Yugoslavia (Received in USA 27 November 1974; received in UK for publication 28 January 1975) In this paper we show that resonance energies and localization energies of benzenoid hydrocarbons can be obtained by counting resonance structures, and that results are tantamount to the best obtainable from highly parameterized SCF-LCAO-MO calculations.

The number of Kekul6 structures for a benzenoid hydrocarbon is accepted to be an approximate empirical measure of resonance stabilization.<sup>1</sup> However, resonance energies are usually estimated as delocalization energies obtainable from HUckel molecular orbital (HMO) or semi-empirical self-consistent-field (SCF) calculations. The extensive variable bond-length SCF results that have been obtained by Dewar and co-workers<sup>2</sup> can be taken to provide a reliable reference set of resonance energies. Recent modified HMO procedures that use open-chain  $\pi$ -systems as reference structures<sup>3,4</sup> give resonance energies for aromatic hydrocarbons that correlate closely with the Dewar SCF results.

A parameterized structure-resonance theory based solely on covalent Kekul6 structures has also been developed that precisely duplicates the Dewar resonance energies.' During the course of that development we noted the correlation between the Kekul6 structure count<sup>6</sup> (SC) and Dewar resonance energy illustrated in the Table. Since the average difference between the SCF resonance energies and those calculated from the algorism is only f0.04 eV, this procedure is a very simple and convenient method to calculate resonance energies and ortho or para localization energies of SCF quality. The quantities so obtained allow one to calculate accurate experimental heats of formation<sup>7</sup> and to correlate various kinds of reactivity including carcinogenicities.<sup>8</sup>

In equations (1) and (2) we give algebraic equalities which give at least a partial understanding of the results delineated in the Table. Equation (1) expresses the fact

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 $\overline{a}$  Reference 6.  $\overline{b}$  Reference 2.

that resonance energies (structure-resonance theory) for  $\pi$ -system composed of aromatic fragments connected by essential single bonds are additive whereas equation (2) shows that the SC

$$
RE(A-B\cdot\cdot\cdot\cdot Z) = RE(A) + RE(B) + \cdot\cdot\cdot\cdot + RE(Z)
$$
 (1)

$$
SC(A-B\cdots Z) = [SC(A)][SC(B)]\cdots [SC(Z)]
$$
 (2)

are multiplicative. The relationship between the In SC and resonance energy is therefore exact for such systems. In SCF calculations, the absence of resonance stabilization through essential single bonds has also been pointed out.<sup>2</sup>

One should note also the relationships given by equations (3) and (4). The square of the SC is the product of the eigenvalues (absolute values) of the HMO adjacency

$$
(sc)^{2} - \prod_{j=1}^{j=N} |x_{j}| \tag{3}
$$

2 ln SC = 
$$
\sum_{j=1}^{j=N}
$$
 ln  $|x_j|$  (4)

matrix,  $9$  equation (3), and the logarithm of the SC is therefore half of the sum of the logarithms of the RMO eigenvalues, equation (4). Incorporating the results given in the Table, equation (4) implies an exact linear equality of the SCF resonance energies and the sum of the logarithms of the HMO eigenvalues, equation (5). Furthermore, since the sum term in equation (4) has been shown to be a function of topological invariants<sup>10</sup> (bonds,

$$
RE_{SCF} = 1.185 \ln SC = 0.5925 \sum_{j=1}^{1=N} \ln |X_j|
$$

atoms, number and types of rings, etc.) the SCF resonance energy is also only a function of these same topological quantities. These latter relationships remain to be clarified. Acknowledgement - We thank the Robert A. Welch Foundation of Houston, Texas for financial support.

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